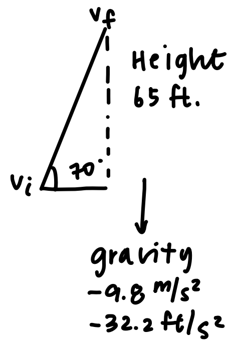
HAGRID'S MOTORBIKE ADVENTURE

Specs:
65 ft spike
70+° angle



Equations

Newtonian Mechanics

$$v = at + v_0$$
$$x = x_0 + v_0 t + \tfrac{1}{2}at^2$$
$$x = x_0 + \tfrac{1}{2}(v + v_0)t$$
$$v^2 = v_0^2 + 2a(x - x_0)$$
$$x = x_0 + v_f t - \tfrac{1}{2}at^2$$

Theoretical calculations:

up length $(x - x_0) = 69.17$ ft.
$v_f = 0$, $v_i = ?$

$a = -32.2 \sin 70$
$\ \ = -30.26\ ft/s^2$

$v^2 = v_0^2 + 2a(x - x_0)$
$0 = v_i^2 + 2(-30.26)(69.17)$

$0 = (-30.26)t + 64.7$

* $v_i = 64.7\ ft/s$ (~44.1 mph)

* $t = 2.138\ s$

From the bottom of the hill.

Assumptions:
- The switch track is directly at the base of the spike.
- The ride vehicle uses the entirety of the height (65 ft) to reach zero velocity.

With these assumptions, the switch track has around 4 seconds.

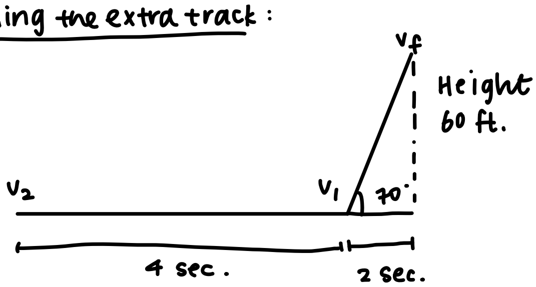
Based on observation, the above assumptions are not entirely accurate.

In reality:
- There is track in between the switch track and the base of the spike.
- The ride vehicle travels most, but not all, of the height.
- Along the track ~~in~~ between the switch track and the spike's base, boosters or speed trimmers can increase or decrease the speed of the ride vehicle.

timed trials
6.40
6.21
6.49
6.25

By observation, there are actually about 6.34 seconds of ride time between the switch track and the spike, or about 12 – 13 seconds for the track to switch.

Adding the extra track:



→ $v_2 = v_1$ because acceleration ≈ 0 without boosters/trimmers, disregarding friction/drag.

$v \cdot t = x - x_0$
$(62.163)(4.34) = 269.787$ ft.

Estimating about 5 feet of spike height as a buffer; thus, $65 - 5 = 60$ ft:

→ up length $(x - x_0) = 63.85$ ft

$v_f^2 = v_0^2 + 2a(x - x_0)$
$0 = v_1^2 + 2(-30.26)(63.85)$
$v_f = at + v_i$
$0 = (-30.26)t + 62.163$

* $v_1 = 62.163\ ft/s$, $t_1 = 2.05$ seconds
(or 43.53 mph)

Theoretically, we would need ~270 ft of track for the ride vehicle to travel for 4 seconds. Realistically, a theme park may not have this space.

If we estimate using different track lengths in between the switch track & spike, $v_1 = 62.163$ ft/s, and with $t = 4$ seconds,

when $x - x_0 = 200$ ft,

$$x - x_0 = \frac{1}{2}(v + v_0)t$$
$$200 = 1/2\,(62.163 + v_2)\cdot 4$$

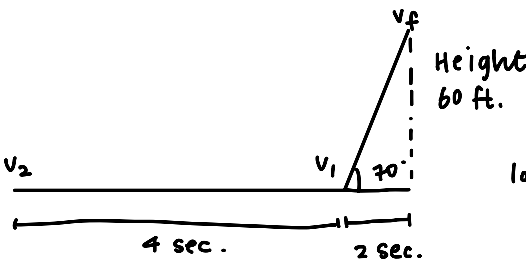
$v_2 = 37.837$ ft/s
$a = 6.08$ ft/s² (or 1.85 m/s²)

when $x - x_0 = 175$ ft,
$v_2 = 25.337$ ft/s
$a = 9.207$ ft/s² (or 2.81 m/s²)

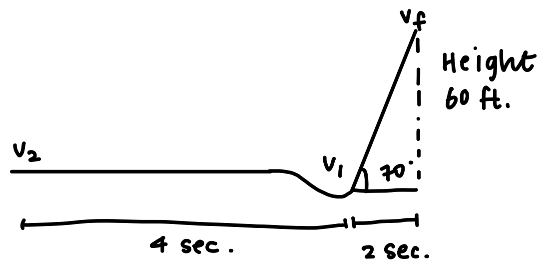
when $x - x_0 = 150$ ft,
$v_2 = 12.837$ ft/s
$a = 12.3315$ ft/s² (or 3.76 m/s²)

Based on the calculated initial velocity (v_2) and the acceleration needed to increase it to the velocity before the spike (v_1), we will use a track extension of 200 ft.

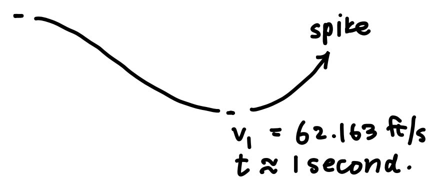
Hagrid's specifically also features a dip before the spike, adding to the velocity of the ride vehicle. So, rather than:



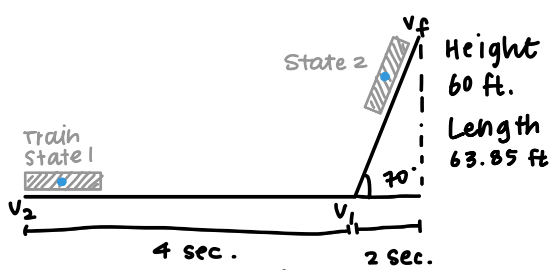
the coaster looks more like:



However, because the starting and ending heights are the same, it does not impact the energy of the vehicle. Thus, in this model, we will not consider this feature.



More Detailed Time Calculations:



Distance: 200 ft.
Acceleration: 6.08 ft/s²

Where the train is no longer a point in space:

Vehicle / Train Length: 33.1 ft.

Because of the train length, the center of mass is at 16.55 ft away from the originally calculated endpoints.

Thus, $(x' - x_0) = 200 - 16.55 = 183.45$ ft. and the new track length for the spike is
$63.85 - 16.55 = 47.3$ ft.

Forwards Total Time:

New Peak Height's Length:
47.3 ft.
$v_f = 0$ ft/s $a = -30.26$ ft/s²

$v_f^2 = v_0^2 + 2a(x - x_0)$
$0 = v_1^2 + 2(-30.26)(47.3)$
$v_f = at + v_i$
$0 = (-30.26)t + 53.5$ ft/s

$v_i = 53.5$ ft/s $t = 1.77$ seconds

New Distance:
183.45 ft.
$v_1 = 53.5$ ft/s $a = 6.08$ ft/s²

$v_1^2 = v_0^2 + 2a(x - x_0)$
$(53.5)^2 = v_0^2 + 2(6.08)(183.45)$
$v_1 = at + v_i$
$53.5 = 6.08t + 25.13$

$v_i = 25.13$ ft/s $t = 4.67$ seconds

Forward Total: 4.67 seconds + 1.77 seconds

Backwards Total Time:

Going Back Down the Hill: 1.77 seconds
Horizontal Track after: $v_i = 53.5$ ft/s $v_f = 53.5$ ft/s $x - x_0 = 183.45$ ft

$x - x_0 = v_0 t + \frac{1}{2}at^2$
$183.45 = 53.5t + 0$

$t = 3.43$ seconds

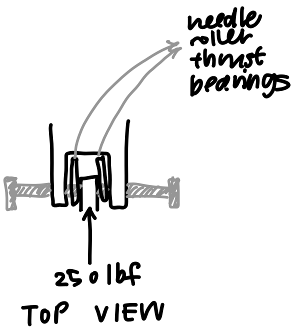
Backwards Total: 1.77 seconds + 3.43 seconds

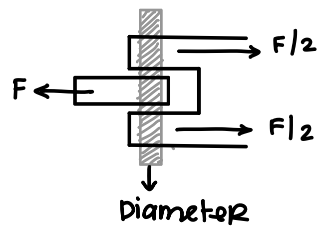
Total Time to switch the Tracks: 11.64 Seconds

→ This is plenty of time to switch the tracks from one configuration to another.

Sizing the Bolts

Adjusting the diameter of the bolts so that the applied force does not exceed the material's shear yield stress.



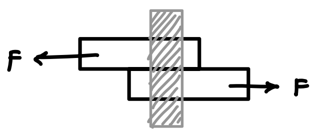


Stress on a Bolt:
Double shear

$$\tau = \frac{F}{2A} = \frac{2F}{\pi D^2}$$

Shear yield stress = 0.75 × Tensile strength

compared to a single shear:



$$\tau = \frac{F}{A}$$

According to McMaster Carr,

High-Strength Steel Tensile Strength = 150.000 psi

shear yield stress = 0.75 × 150,000 = 112500 psi

<units> $psi = \frac{lbf}{in^2}$

Iteration #1: 1.5" diameter

$$\tau = \frac{2F}{\pi D^2} = \frac{2 \cdot 250\ lbf}{\pi \cdot (1.5\ in)^2} = \frac{500}{1.5^2 \cdot \pi} = 70.735\ \frac{lbf}{in^2} = 70.735\ psi$$

Safety Factor:

$$\text{Safety Factor} = \frac{\text{capacity (resisting force)}}{\text{demand (disturbing force)}} = \frac{112500}{70.735} = 1590.44$$

∴ 1.5" diameter rod made of high strength steel is overengineered.

Iteration #2: 0.25" diameter

$$\tau = \frac{2 \cdot 250\ lbf}{\pi \cdot (0.25\ in)^2} = \frac{500}{0.25^2 \cdot \pi} = 2546.48\ psi$$

Safety Factor:

$$F = \frac{C}{D} = \frac{112500}{2546.48} = 44.18$$

∴ The original diameter of 0.25" works. as the safety factor reflects that the rod will not fail.

ASTM calculations / considerations

ASTM F2291 - 22a : Standard Practice for Design of Amusement Rides and Devices

8.22 Load combinations for Strength using ASD (Allowable Stress Design):

8.22.1 The following loads are to be considered:

Dead Load: permanent load due to the weight of the structural elements and the permanent features on the structure.

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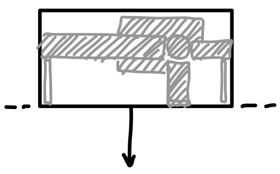
design iteration #1 :

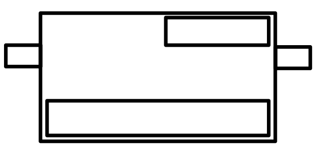


gravity and body force:

currently with Steel Sel as material for all components

mass = 11727.78 lbs. = 5.86 U.S. tons.

design iteration #2 : after reducing material through structural analysis



total assembly original weight:

11727.78 lbs.

original base axis mass:

3327252.67 g = 7335.34 lbs.

iteration #2 base axis mass:

1983955.73 g = 4373.87 lbs.

new assembly weight :

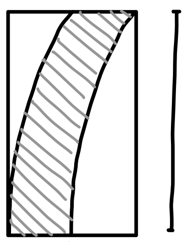
11727.78 - 7335.34 + 4373.87 = 8766.31 lbs.

= 4.383 U.S. Tons.

7.1.4. Sustained acceleration duration limits are shown in this section (see Fig. 6-8).
The following definitions apply:
7.1.4.1 Acceleration units are "g" (32.2 ft/s/s or 9.81 m/s/s)

And
7.1.5. Simultaneous combinations of single axis accelerations shall be limited as follows:
7.1.5.1 The instantaneous combined acceleration magnitude of any two axes shall be limited by a curve that is defined in each quadrant by an ellipse. The ellipse is centered at (0,0) and is characterized by major and minor radii equal to the allowable 200 ms g limits. Graphical representations of this requirement are presented in figs. 10-17.

Switch Track Length: 144 ft. → y-direction Acceleration for
Vehicle speed: 25.13 ft/s → 5.7 seconds



144 ft

curved track

centripetal acceleration:

$$a_c = \frac{v^2}{r} \quad \text{where } v = 25.13 \text{ ft/s}, \; r = 340 \text{ in} = 28.33 \text{ ft}$$

$$\therefore a_c = \frac{25.13^2}{28.33} = 22.29 \text{ ft/s}^2 = 0.693 \text{ G}$$

in the Y direction

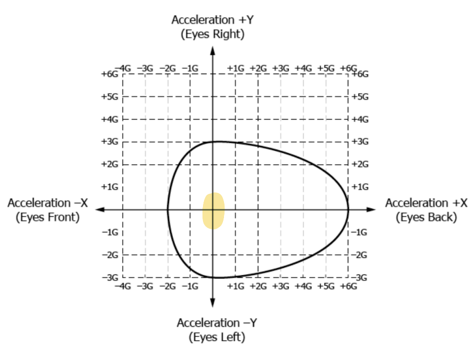


FIG. 10 Allowable Combined Magnitude of X and Y Accelerations

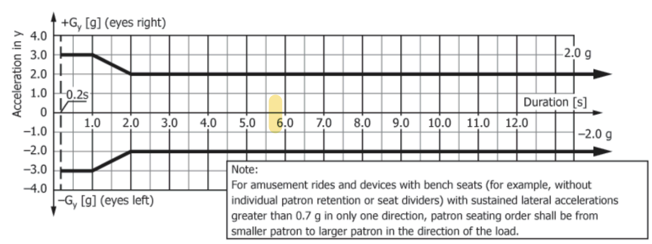


FIG. 7 Acceleration-Duration Limits for Gy (Eyes Right and Eyes Left)

The acceleration does not exceed the limits set by the standards.

8.3 35,000 operational Hour criteria:

8.3.1 All primary structures of an amusement ride or device (for example, tracks, columns, hubs, and arms) shall be designed using calculations and analyses that are based on the minimum 35,000 operational hour criteria. The designer/engineer shall verify the calculations and analyses meet or exceed this minimum operational hour requirement. This requirement is intended to ensure that all primary structures within an amusement ride or device are designed for at least a minimum fatigue life.

General Reduction for Load / unload Time:

$$\frac{(\text{Total Load/Unload Time for one Ride cycle})}{\left(\text{Total Load/Unload Time for One Ride cycle}\right) + \left(\text{Time for one ride cycle}\right)}$$

Operational Hours:

$$\text{35000 operational Hours criteria} \times \left(1.00 - \text{general reduction for Load/unload Time}\right)$$

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*Note: Estimation

Aiming for 2000 riders per hour:

2000 riders per hour = > 33 riders per minute

With 12 riders per train, we need 2.78 trains to leave every minute.

→ we need a train to leave every 21.6 seconds.

Time for one ride cycle: 2.5 minutes = 150 seconds

Load/unload Time for one ride cycle: 21.6 × 2 = 43.2 seconds

→ general reduction of Load/unload Time:

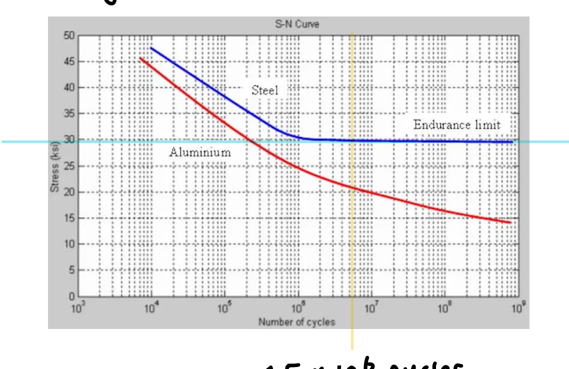
$$\frac{43.2}{43.2 + 150} = 0.2236$$

Operational Hours: 35000 × (1 − 0.2236) = 27174 Hours

Number of cycles based on operational Hours:

$$27174 \text{ Hours} \times \frac{1 \text{ train}}{21.6 \text{ seconds}} = 4528667 \text{ cycles}$$ (as defined by materials engineering)

Fatigue Life of Steel



4,528,667 cycles = 4.5×10^6 cycles

Stress based on operational hours:

29 ksi = 199947961.5 N/m^2

$\approx 2.0 \times 10^8$ N/m^2

The simulation stress must remain under this value.

Structural Simulations & Finite Element Analysis (FEA)

Determining How much Force to calculate with:

Curved Track: 116192.76 grams
= 1139.46 Newtons

Straight Track: 111943.59 grams
= 1097.79 Newtons

Average Roller coaster Train: 500 kg when fully loaded with passengers
= 4903.325 Newtons

Total maximum: 1139.46 + 4903.325 = 6042.785 Newtons
Mass of Base Axis: 3327252.67 grams

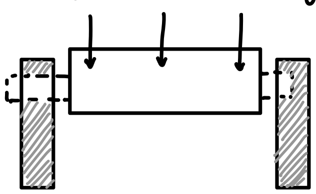
Yield strength of material : 6.204×10^8 N/m²

Operational stress from Above : 2.0×10^8 N/m²

Design iteration #1:

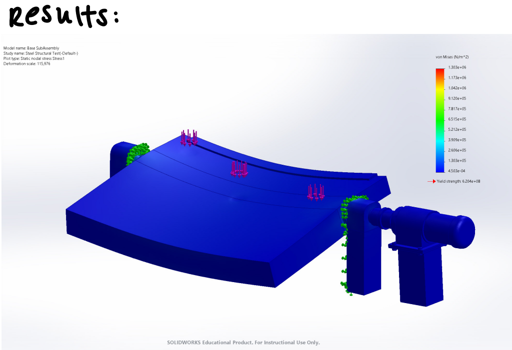
Simulation #1:

3 External Loads on Base Subassembly



Fixed: Base Axis Supports

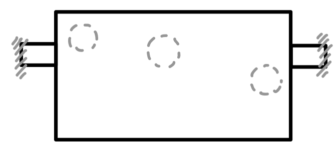
Results:



maximum stress: 1.303×10^6 N/m²

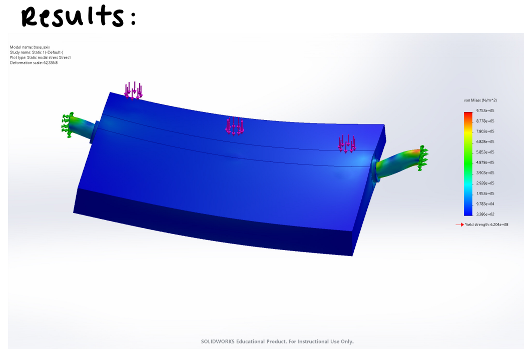
Simulation #2:

3 External Loads on Base Axis component



Fixed: Planes of each shaft end

Results:

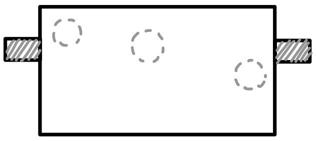


maximum stress: 9.753×10^5 N/m²

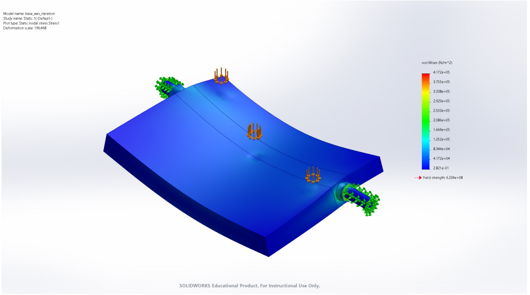
Based on the stress points of these results, the areas connecting the shaft and the rotating platform were thickened.

simulation #3: most "realistic" simulation

3 External Loads on
Base Axis component



Fixed: center shaft



maximum stress on the material:
4.172×10^5 N/m²

material
Safety Factor: $\frac{2.0 \times 10^8}{4.172 \times 10^5}$ = 479.38 → over-engineered.

Even simulation #1, which produced the largest stress at 1.303×10^6 N/m²,
has a safety factor of 153.49 (still overengineered).

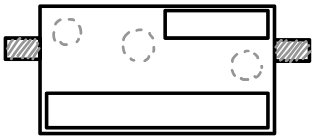
Because this configuration is overengineered in all simulations, the next iterations attempt to reduce the amount of material, which would lower the cost and weight.

The highest points of stress are at the axis points. Thus, since the center face is not impacted, material can be reduced in these areas.

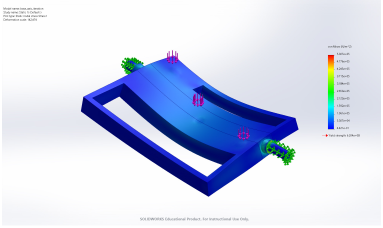
Design criteria: must avoid removing material from the points of external force, and must maintain the geometry (width / length).

Iteration #2:

Decreasing material in the z-direction via cut-outs.



Fixed: center shaft

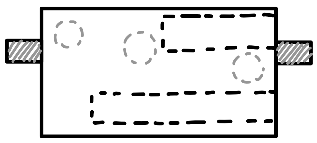


maximum stress: 5.307×10^5 N/m²

There isn't too much of a difference in the stress.
mass = 1983955.73 grams → 40.37% decrease in mass

Iteration #3:

Decreasing material in the y-direction by making the rotating platform partially hollow.



Fixed: center shaft

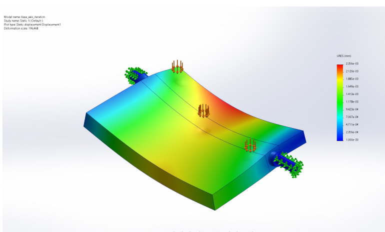
The stress results were lower than that of Iteration #2.

mass = 208869.07 grams

→ 37.22% decrease in mass

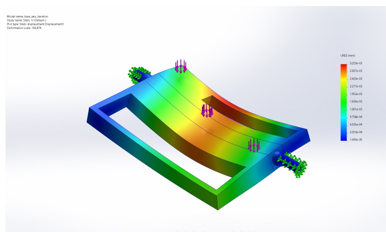
Comparing Displacements

Iteration #1:



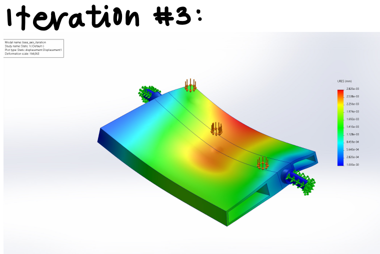
2.356×10^{-3} mm

Iteration #2:



3.253×10^{-3} mm

Iteration #3:



2.820×10^{-3} m

Displacements:

#1: 9.28×10^{-5} in

#2: 1.28×10^{-4} in

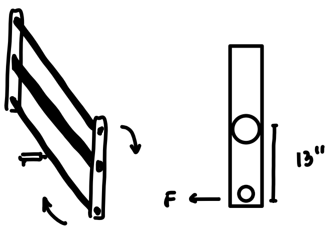
#3: 1.11×10^{-4} in

Although #2 has the largest displacement, the values are not large enough to impact the structure.

Thus, if we are prioritizing the least amount of mass, Iteration #2 would be the best option.

Moving forward, I would use this model to iterate different geometries that would further decrease the mass and produce different stress and displacement results.

Calculating Forces for the Locking mechanism



mass of entire subassembly : 960465. 31 grams
mass of rods & shaft : 929136. 59 grams

Torque : $\tau = r F \sin\theta$
where $\theta = 90^\circ$ and $r = 13" = 0.3302$ m

$F = (960.46531\ kg) \cdot (9.81\ m/s^2)$
$= 9422.165\ N = 2118.187$ lbs

$\tau = (960.46531\ kg) \cdot (9.81\ m/s^2) \cdot (0.3302\ m)$
$= 3111.2$ N·m of Torque Needed.

Currently in my model, the actuator has a dynamic push/pull load of 250 lbs.
Possible solutions :

- Replace the actuator with an actuator with a larger dynamic push/pull load.
- Redesign the locking mechanism rod to accommodate for multiple actuators, and configure them to operate simultaneously.
- Keep the single actuator, but add a motor to the locking mechanism so that there is less torque needed to rotate the assembly.